
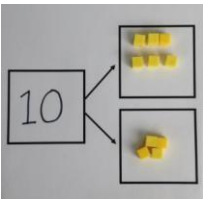

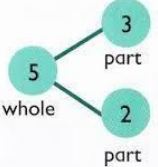

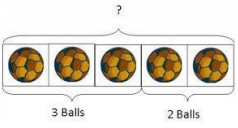
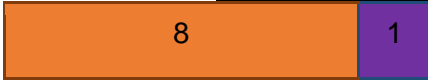
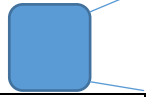

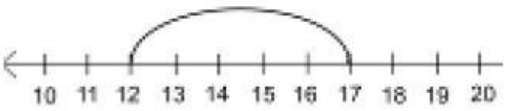




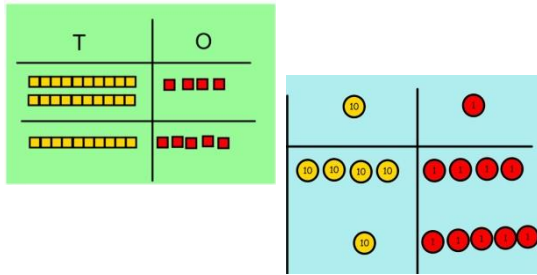
Addition

Objective and Strategies	Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole: part-whole model</p>	   <p>Use cubes to add two numbers together as a group or in a bar.</p>	    <p>Use pictures to add two numbers together as a group or in a bar.</p>	<p>$4 + 3 = 7$</p> <p>$10 = 6 + 4$</p>  <p>Use the part-part whole diagram as shown above to move into the abstract.</p>
<p>Starting at the bigger number and counting on</p>	 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	<p>$12 + 5 = 17$</p>  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	<p>$5 + 12 = 17$</p> <p>Place the larger number your head and count on the smaller number to find your answer.</p>

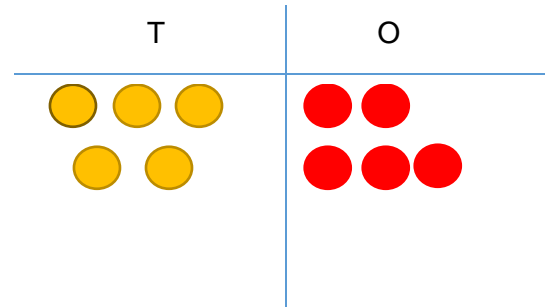
Column method- no regrouping

$24 + 15 =$

Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.



After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.



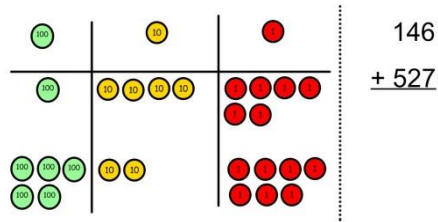
Calculations

$21 + 42 =$

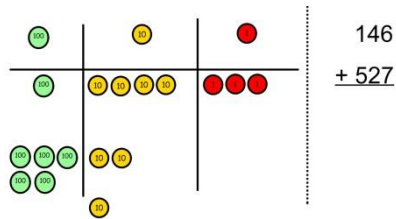
$$\begin{array}{r} 21 \\ + 42 \\ \hline \end{array}$$

Column method-regrouping

Make both numbers on a place value grid.



Add up the units and exchange 10 ones for one 10.

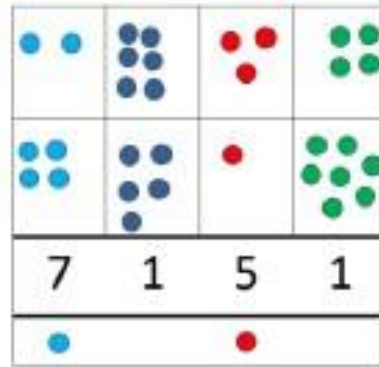


Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.



Start by partitioning the numbers before moving on to clearly show the exchange below the addition.

$$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ \hline 60 + 13 = 73 \end{array}$$

$$\begin{array}{r} 536 \\ + 85 \\ \hline \end{array}$$

As the children move on, introduce

$$\begin{array}{r} 621 \\ 11 \\ \hline \end{array}$$

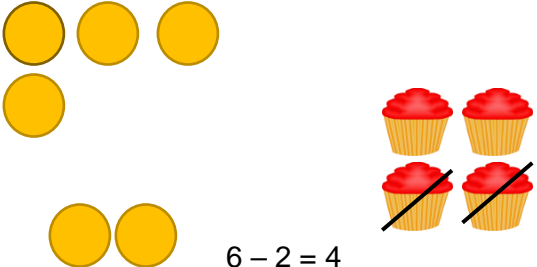
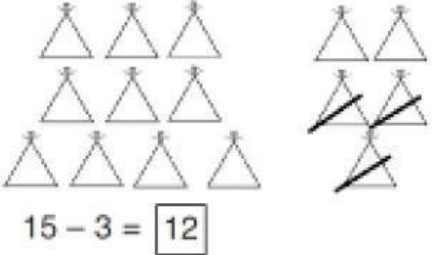


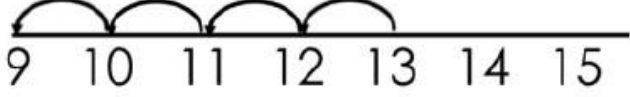
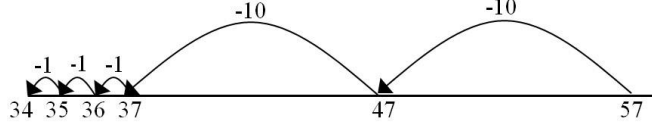
decimals with the same number of decimal places and different. Money can be used here.

$$\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \end{array}$$

$$\begin{array}{r} \pounds 23.59 \\ + \pounds 7.55 \\ \hline \pounds 31.14 \end{array}$$

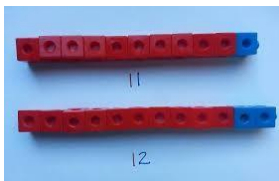
$$\begin{array}{r} 23.361 \\ 9.080 \\ + 1.300 \\ \hline 93.511 \\ 212 \end{array}$$

Subtraction

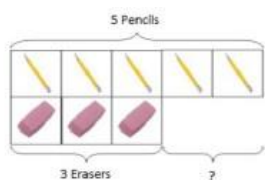
Objective and Strategies	Concrete	Pictorial	Abstract
<p>Taking away ones</p>	<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p>  <p>$6 - 2 = 4$</p>	<p>Cross out drawn objects to show what has been taken away.</p>  <p>$15 - 3 = 12$</p>	<p>$18 - 3 = 15$</p> <p>$8 - 2 = 6$</p>
<p>Counting back</p>	<p>Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.</p>  <p>$13 - 4$</p> <p>Use counters and move them away from the group as you take them away counting backwards as you go.</p> 	<p>Count back on a number line or number track</p>  <p>Start at the bigger number and count back the smaller number showing the jumps on the number line.</p>  <p>This can progress all the way to counting back using two 2 digit numbers.</p>	<p>Put 13 in your head, count back 4. What number are you at? Use your fingers to help.</p>

Find the difference

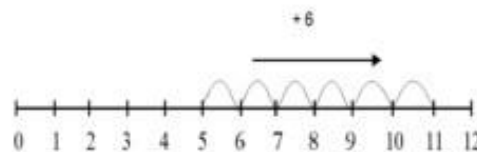
Compare amounts and objects to find the difference.



Use cubes to build towers or make bars to find the difference



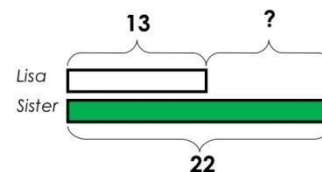
Use basic bar models with items to find the difference



Count on to find the difference.

Comparison Bar Models

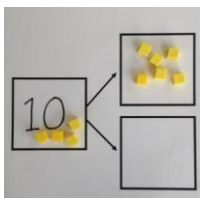
Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.



Draw bars to find the difference between 2 numbers.

Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.

Part Part Whole Model

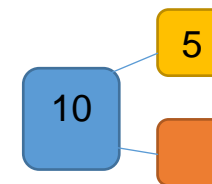
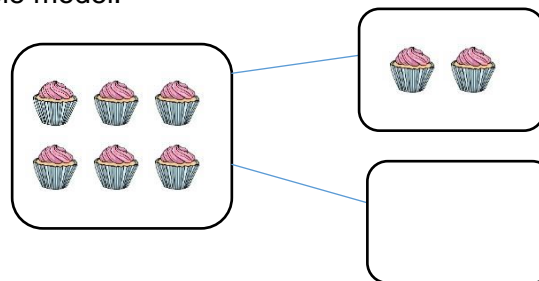


Link to addition- use the part whole model to help explain the inverse between addition and subtraction.

If 10 is the whole and 6 is one of the parts. What is the other part?

$$10 - 6 =$$

Use a pictorial representation of objects to show the part part whole model.



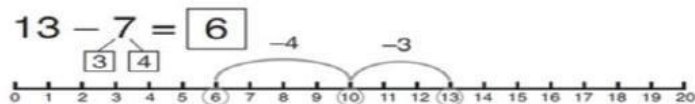
Move to using numbers within the part whole model.

Make 10

$14 - 9 =$



Make 14 on the ten frame. Take away the four first to make 10 and then take away one more so you have taken away 5. You are left with the answer of 9.



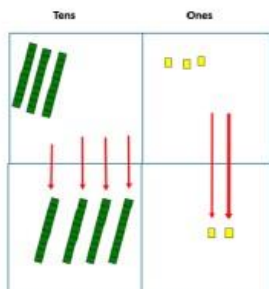
Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.

$16 - 8 =$

How many do we take off to reach the next 10?

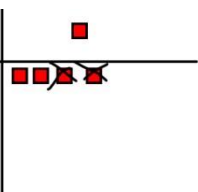
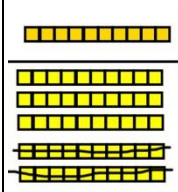
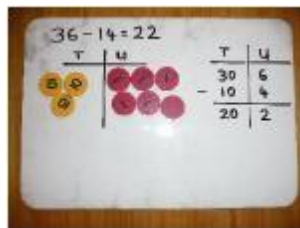
How many do we have left to take off?

Column method without regrouping



Use Base 10 to make the bigger number then take the smaller number away.

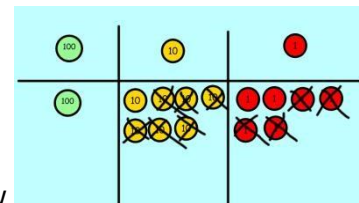
Show how you partition numbers to subtract. Again make the larger number first.



Calculations

$$\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$$

Draw the Base 10 or place value counters alongside the written calculation to



Calculations

$$\begin{array}{r} 176 \\ - 64 \\ \hline 112 \end{array}$$

help to show working.

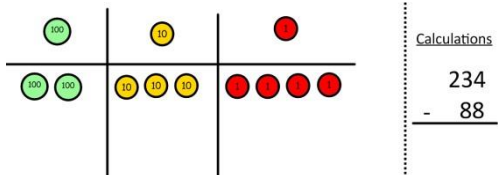
$$\begin{array}{r} 47 - 24 = 23 \\ 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$$

This will lead to a clear written column subtraction.

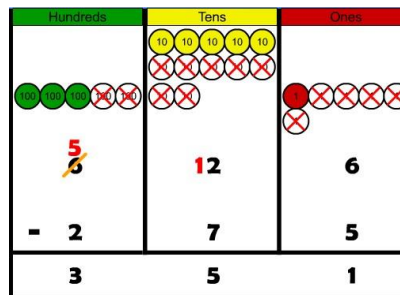
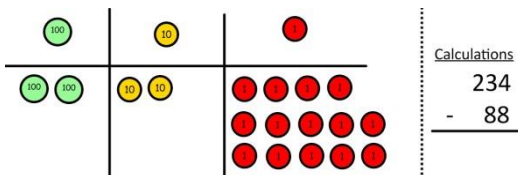
Column method with regrouping

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

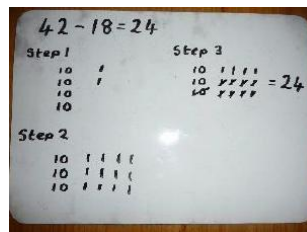
Make the larger number with the place value counters



Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.

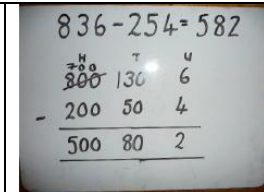


Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

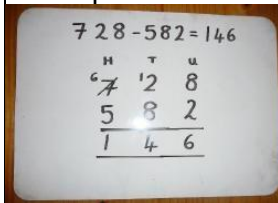


When confident, children can find their own way to record the exchange/regrouping.

Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.



Children can start their formal written method by partitioning the number into clear place value columns.



Moving forward the children use a more compact method.

Now I can subtract my ones.

Calculations

$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.

Calculations

$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

Now I can take away eight tens and complete my subtraction

Calculations

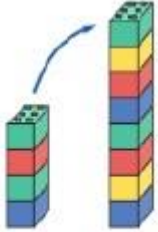

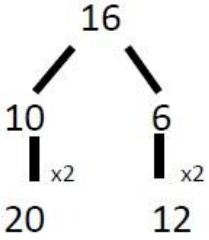
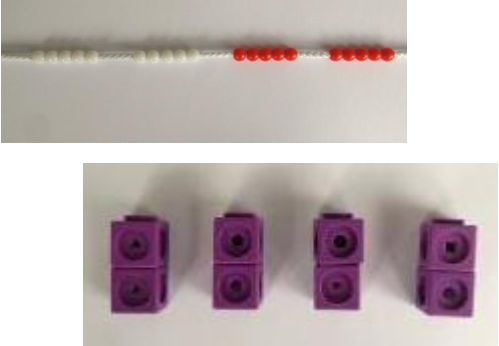
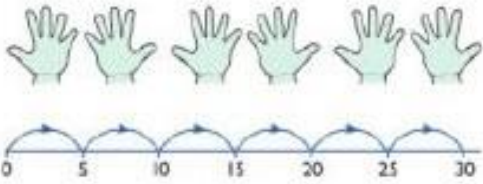
$$\begin{array}{r} \cancel{234} \\ - 88 \\ \hline 146 \end{array}$$

Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

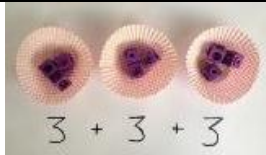
This will lead to an understanding of subtracting any number including decimals.

$$\begin{array}{r} 5 \quad 12 \quad 1 \\ 2 \quad \cancel{6} \quad \cancel{3} \quad . \quad \cancel{0} \\ - \quad 2 \quad 6 \quad . \quad 5 \\ \hline 2 \quad 3 \quad 6 \quad . \quad 5 \end{array}$$

Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
<p>Doubling</p>	<p>Use practical activities to show how to</p>  <p>double 4 is 8 $4 \times 2 = 8$</p> <p>double a number.</p>	<p>Draw pictures to show how to double a number.</p> <p>Double 4 is 8</p> 	 <p>Partition a number and then double each part before recombining it back together.</p>
<p>Counting in multiples</p>	 <p>Count in multiples supported by concrete objects in equal groups.</p>	 <p>Use a number line or pictures to continue support in counting in multiples.</p>	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>

Repeated addition

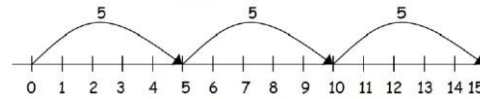


Use different objects to add equal groups.

There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?



2 add 2 add 2 equals 6



$$5 + 5 + 5 = 15$$

Write addition sentences to describe objects and pictures.



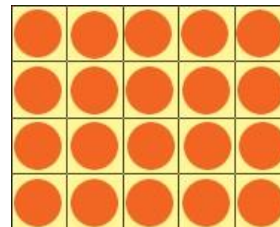
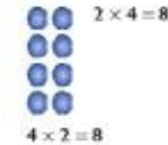
$$2 + 2 + 2 + 2 + 2 = 10$$

Arrays- showing commutative multiplication

Create arrays using counters/ cubes to show multiplication sentences.

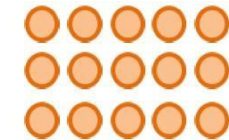


Draw arrays in different rotations to find **commutative** multiplication sentences.



Link arrays to area of rectangles.

Use an array to write multiplication sentences and reinforce repeated addition.



$$5 + 5 + 5 = 15$$

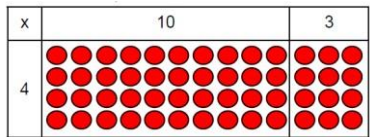
$$3 + 3 + 3 + 3 + 3 = 15$$

$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

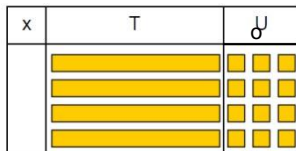
Grid Method

Show the link with arrays to first introduce the grid method.



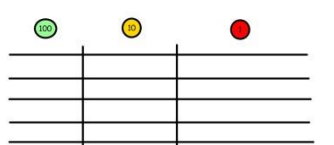
4 rows of 10
4 rows of 3

Move on to using Base 10 to move towards a more compact method.



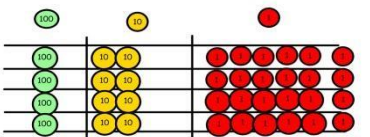
4 rows of 13

Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.



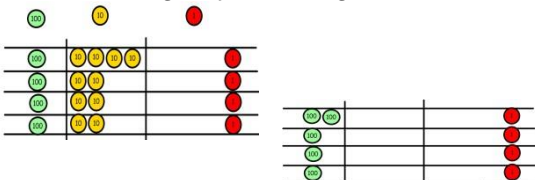
Calculations
 4×126

Fill each row with 126.



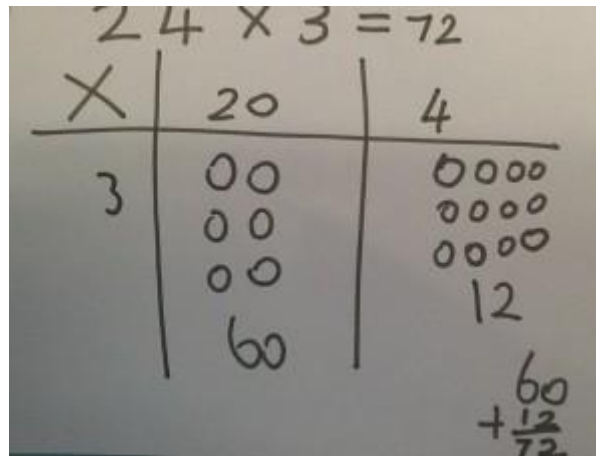
Calculations
 4×126

Add up each column, starting with the ones making any exchanges needed.



Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.

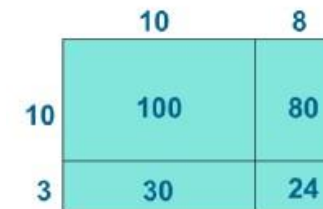


Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

x	30	5
7	210	35

$$210 + 35 = 245$$

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

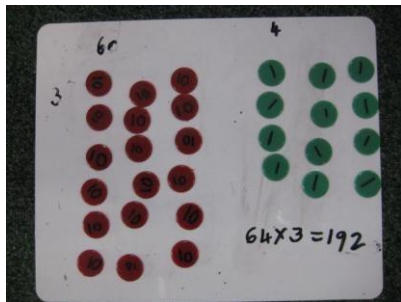


x	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

Then you have your answer.

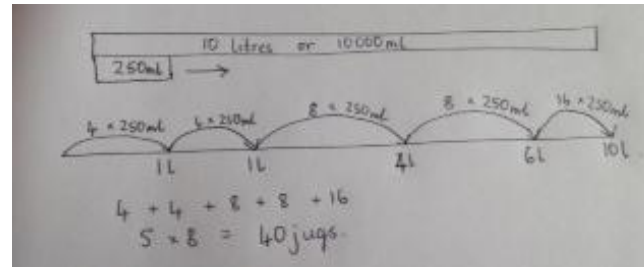
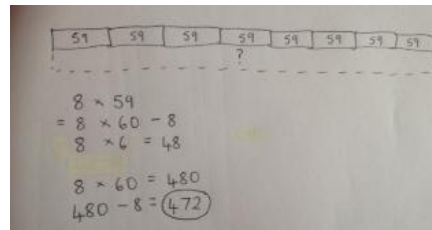
Column multiplication

Children can continue to be supported by place value counters at the stage of multiplication.



It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.



Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

If it helps, children can write out what they are solving next to their answer.

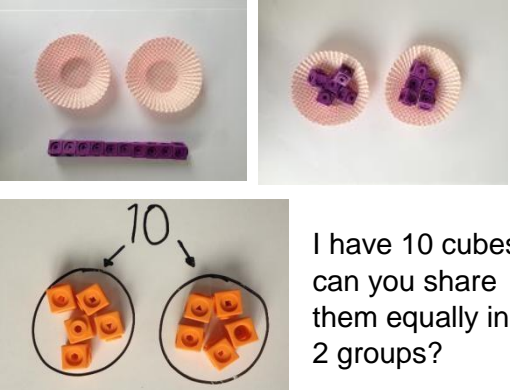
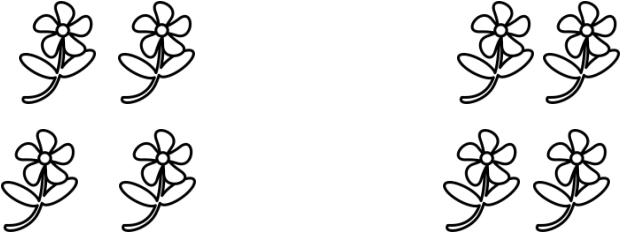
$$\begin{array}{r}
 32 \\
 \times 24 \\
 \hline
 8 \quad (4 \times 2) \\
 120 \quad (4 \times 30) \\
 40 \quad (20 \times 2) \\
 600 \quad (20 \times 30) \\
 \hline
 768
 \end{array}$$

$$\begin{array}{r}
 74 \\
 x 63 \\
 \hline
 12 \\
 210 \\
 240 \\
 + 4200 \\
 \hline
 4662
 \end{array}$$

This moves to the more compact method.

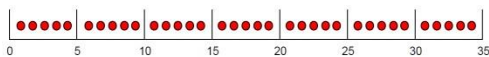
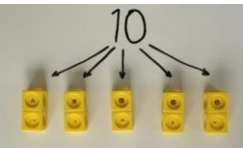
$$\begin{array}{r}
 2 3 1 \\
 1342 \\
 \times 18 \\
 \hline
 13420 \\
 10736 \\
 \hline
 24156 \\
 1
 \end{array}$$

Division

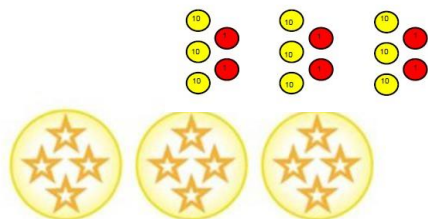
Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	 <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  $8 \div 2 = 4$	<p>Share 9 buns between three people.</p> $9 \div 3 = 3$

Division as grouping

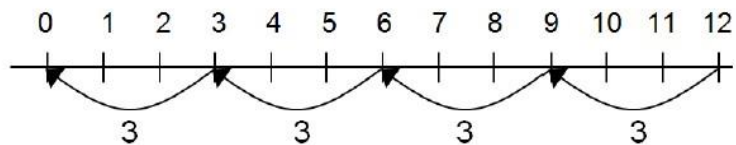
Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.



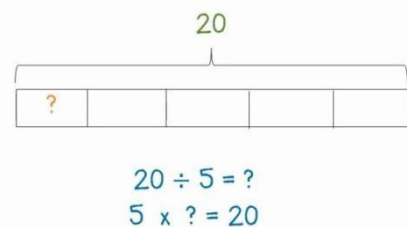
$$96 \div 3 = 32$$



Use a number line to show jumps in groups. The number of jumps equals the number of groups.



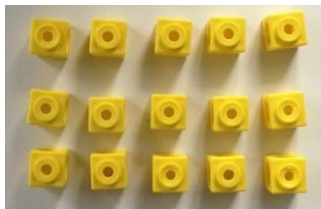
Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.



$$28 \div 7 = 4$$

Divide 28 into 7 groups. How many are in each group?

Division within arrays

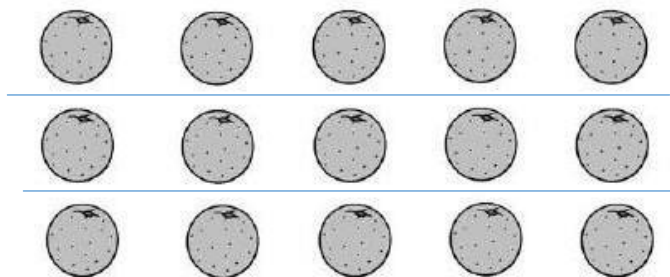


Link division

to multiplication by creating an array and thinking about the

number sentences that can be created.

Eg $15 \div 3 = 5$ $5 \times 3 = 15$
 $15 \div 5 = 3$ $3 \times 5 = 15$



Draw an array and use lines to split the array into groups to make multiplication and division sentences.

Find the inverse of multiplication and division sentences by creating four linking number sentences.

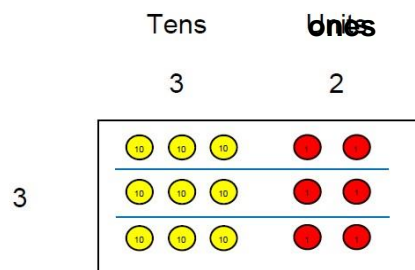
$$7 \times 4 = 28$$

$$4 \times 7 = 28$$

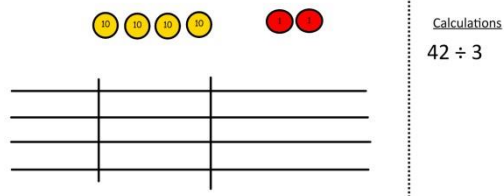
$$28 \div 7 = 4$$

$$28 \div 4 = 7$$

Short division

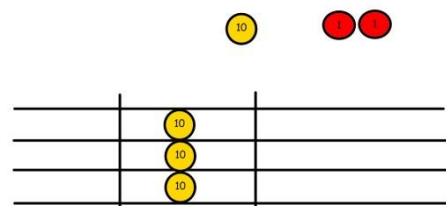


Use place value counters to divide using the bus stop method alongside

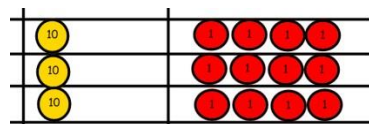


$$42 \div 3 =$$

Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.

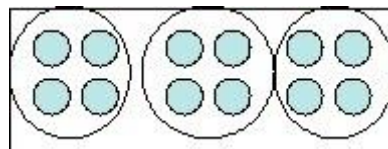


We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much in 1 group so the answer is 14.

Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 218 \\ 3 \overline{) 872} \end{array}$$

Move onto divisions with a remainder.

$$\begin{array}{r} 86 \text{ r } 2 \\ 3 \overline{) 432} \end{array}$$

Finally move into decimal places to divide the total accurately.

$$\begin{array}{r} 430.5 \div 35 \\ 012.3 \\ 5 \overline{) 061.5} \\ 7 \overline{) 430.5} \end{array}$$

If the number is not prime... 'Factorise' 35 into 5 and 7. Divide the question by 5 and then that answer by 7.

If the number is prime... use formal short division method, dividing by the two digit number.



Claypole Church of England Primary School

CALCULATION POLICY

Reviewed	February 2022
Next Review Date	February 2023
Adopted	February 2022

What is the purpose of the policy?

The purpose of this calculation policy is to ensure consistency and progression in the teaching of the different calculation methods across the school. It aims to give an overview of the key written calculation strategies that will be taught in all year groups. All members of staff are expected to be familiar with this policy and apply it consistently across the school.

How is the calculation policy set out?

This calculation policy has been organised by year group, considering the national curriculum 2014 expectations. Each page refers to a different operation (addition, subtraction, multiplication and division) and shows a progression in calculation from the Foundation Stage up to Year Six.

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly. These skills then lead on to more formal written methods of calculation.

It is important that children do not abandon jottings and mental methods once formal written methods are introduced. Therefore children should always be encouraged to look at a calculation/problem and then decide which is the best method to choose. As children become more confident with their calculation, they need to use the following steps when approaching problems:

- 'Can I solve this problem in my head and use a mental method?'
- 'Do I need to use some written jottings to help me?'
- 'Do I need to use more formal written method to solve the problem?'

Concrete, Pictorial, Abstract Approach

The calculation policy is laid out in accordance with one of the key learning principles behind Maths Mastery, the Singapore approach at White Rose maths, as supported by the NCTEM. It supports how we teach maths, with challenge for all at the heart. The approach we use is concrete, pictorial and abstract (often referred to as the CPA approach). The concrete, pictorial, abstract approach, based on research by psychologist Jerome Bruner, suggests that there are three steps (or representations) necessary for pupils to develop understanding of a concept. Reinforcement is achieved by going back and forth between these representations.

Concrete representation

The active stage - a student is first introduced to an idea or a skill by acting it out with real objects. In division, for example, this might be done by separating apples into groups of red ones and green ones or by sharing 12 biscuits amongst 6 children. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

Pictorial representation

The iconic stage - a student has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or picture of the problem. In the case of a division exercise this could be the action of circling objects.

Abstract representation

The symbolic stage - a student is now capable of representing problems by using mathematical notation, for example: $12 \div 2 = 6$.

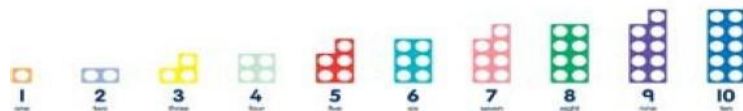
Whilst this calculation policy aims to show the CPA approach to the different calculations, it is not always noted further up the year groups. However, it is expected that the CPA approach is used continuously in all new learning and calculations even when not noted.

This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added. It is a working document and will be revised and amended as necessary.

Addition

EYFS

Before children can move on to the methods for addition they need to be able to count reliably, including one to one correspondence:



5 plus 3 = 8
3 more than 5 = 8

Children need to be able to count on from any number, combining two groups:


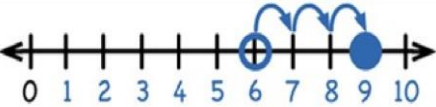
Children will begin to know addition facts to 10:



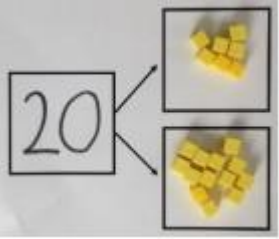
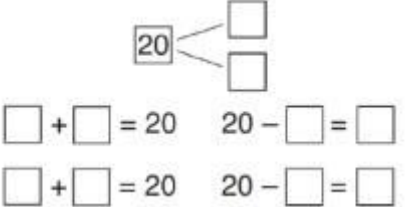

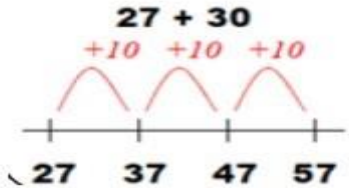
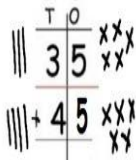
Numicon is used as a primary resource to support the teaching of addition where children are encouraged to visualise the numicon patterns and calculate by combining/partitioning the patterns rather than resort to counting. Practical experiences and activities are at the heart of developing mathematical concepts.

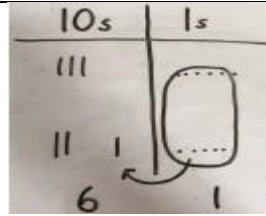
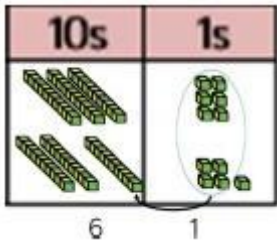
Year 1

Strategies	Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole using the part-part-whole model.</p>		<p>Use pictures to add two groups together in a bar or in a group.</p>	<p>Use part-part-whole diagram as shown below to move into the <u>Use part-part-whole model</u>.abstract.</p> <p>Use cubes to add two numbers together as a group or in a bar. Numicon can also be used. $2 + 3 = 5$</p>

<p>Starting at the bigger number and counting on.</p>	<p>Start with the larger number on the bead string and then count on to the smaller</p>  <p>number 1 by 1 to find the answer.</p>	<p>Start at the larger number on the number line and count on in ones to find the answer.</p> <p>$6 + 3 = 9$</p> 	<p>Place the larger number in your head and count on the smaller number to find your answer.</p> <p>$6 + 3 = 9$</p>
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Year 2

Strategies	Concrete	Pictorial	Abstract
<p>Use known number facts (part-part-whole method).</p>	<p>Children explore ways of making numbers within 20.</p> 	 <p>Use part-part-whole diagram.</p>	<p>$\square + 1 = 16$ $16 - 1 = \square$</p> <p>$1 + \square = 16$ $16 - \square = 1$</p>
<p>Add a 2 digit number and tens.</p>	<p>Use diennes and investigate the pattern (the ones stay the same).</p>  <p>$25 + 10 = 35$</p>	<p>Use number lines to count on in tens alongside concrete resources.</p> <p>$27 + 30$</p> 	<p>$27 + 10 = 37$</p> <p>$27 + 20 = 47$</p> <p>$27 + \square = 57$</p> 
<p>Add two 2-digit numbers</p>	<p>Model using dienes and a place value mat.</p>	<p>Children to represent dienes using lines for 10's and dots or crosses for ones.</p>	<p>Expanded Column Addition:</p> <p>partitioned numbers are written under one another:</p>



$$47 = 40 + 7$$

$$+76 \quad 70 + 6$$

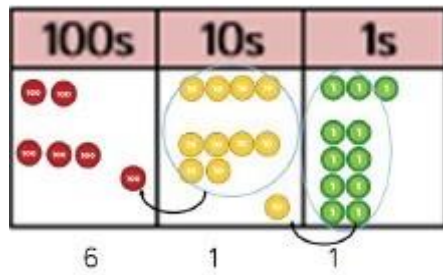
$$110 + 13 = 123$$

Year 3

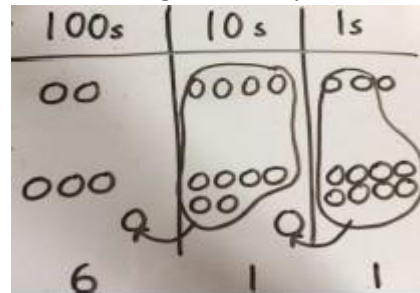
Strategies	Concrete	Pictorial	Abstract
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Add numbers with up to three digits, using formal written methods of columnar addition.

Use of place value counters. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.



Children to represent the counters in a place value chart, circling when they make an exchange.



Compact Column Addition

789 + 642 becomes

$$\begin{array}{r} 789 \\ + 642 \\ \hline 1431 \end{array}$$

Answer: 1431

Add the units first, carry numbers underneath the bottom line, remind pupils of actual value e.g.

Year 4

Strategies	Concrete	Pictorial	Abstract
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Add whole numbers with more than 4 digits, including using formal

Continue to use dienes and place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.
2,334 + 1,123 =

Draw representations using place value diagram.

Continue from previous work to carry hundreds and tens. Relate to money and measures.

written methods (columnar addition).			$\begin{array}{r} \pounds 32.50 + \pounds 21.75 = \pounds 54.25 \\ \pounds 32.50 \\ + \pounds 21.75 \\ \hline \pounds 54.25 \end{array}$
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Year 5

Strategies	Concrete	Pictorial	Abstract
Add numbers with more than 4 digits. Add decimals with decimal places, including money.	Introduce decimal place value counters and model exchange for addition. 	Draw representations using PV grid. $2.37 + 81.79$ 	Tenths, hundredths and thousandths should be correctly aligned, with the decimal point aligned vertically including in the answer. Empty decimal places can be filled with the zero to show the place value of each column.

Year 6

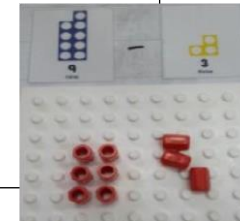
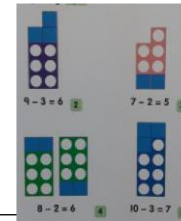
Strategies	Concrete	Pictorial	Abstract
Add several	As year 5		

numbers
of
increasing
complexity.

SUBTRACTION EYES

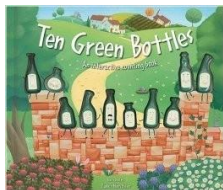
Before children can move onto the methods for subtraction they need to be able to count reliably including one to one, be able to count up or back any number and begin to understand subtraction as take away. This will be supported with these concepts through singing, developing ways of calculations using pictures or apparatus, such as


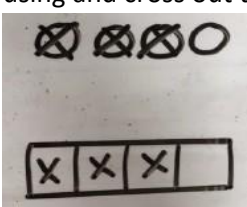
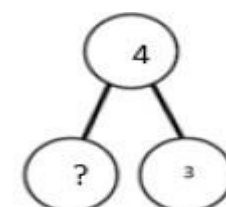
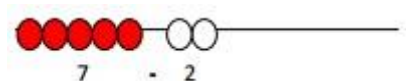
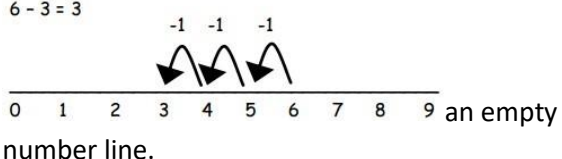
Relate subtraction to finding how many are left when some

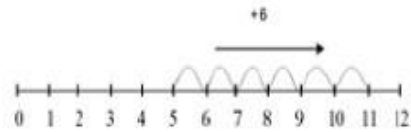


Children will begin to count backwards in familiar contexts, number rhymes and

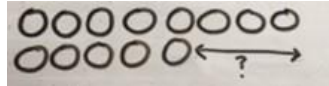
Numicon is used as a primary resource to support the teaching of mathematical concepts. Practical experiences and activities are at the heart of mathematical learning.



Year 1			
Strategie s	Concrete	Pictorial	Abstract
Taking away ones.	<p>Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags</p>  <p>could be used).</p>	<p>Children to draw the concrete resources they are using and cross out the correct amount.</p> 	<p>$4 - 3 = 1$</p>  <p>Use Part-partwhole diagram to link with addition.</p>
Starting at the bigger number and counting back.	<p>Move the beads along the bead string as you count backwards.</p> 	<p>Start at the larger number on the number line and count back in ones to find the answer. Encourage children to progress to</p> <p>$6 - 3 = 3$</p>  <p>an empty number line.</p>	<p>Place the larger number in your head and count back the smaller number to find your answer.</p> <p>$6 - 3 = 3$</p>
Find the difference.	<p>Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).</p> <p>Calculate the difference between 8 and 5.</p>	<p>Count on using a number line to find the difference.</p>	<p>Find the difference between 8 and 5.</p> <p>$8 - 5$, the difference is _____.</p> <p>Children to explore why: $9 - 6 = 8 - 5 = 7 - 4$ have the same</p>



Or children can draw the cubes they have used to illustrate what they need to



calculate.

difference.

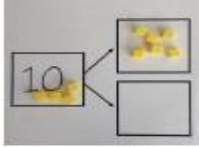
Year 2

Strategies

Concrete

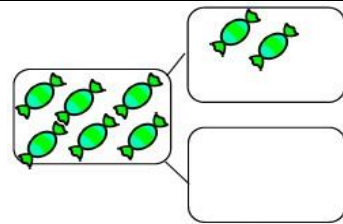
Represent and use number bonds and related subtraction facts within 20.

Link to addition. Use the Part-part-whole model to model the inverse. If 10 is the whole and 6 is one of the parts. What is the other part?

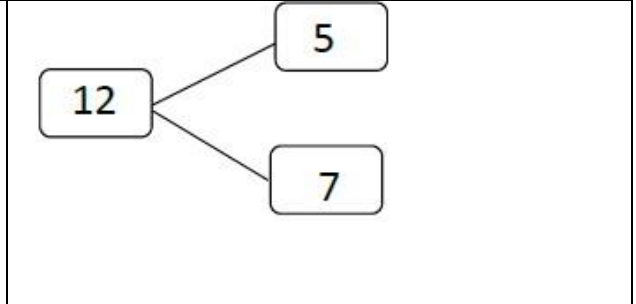


Pictorial

Use part-part-whole diagram.



Abstract



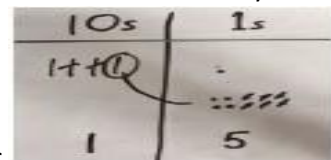
Partitioning to subtract without regrouping

Use dienes to show how to partition the number when subtracting without regrouping.

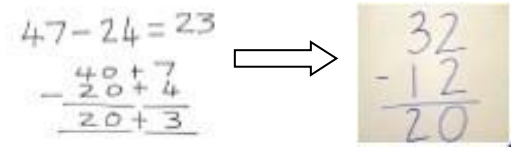


$34 - 13 = 21$

Children draw representations of dienes using lines for tens and dots or crosses for ones. They then cross off the smaller amount.



Expanded Column Subtraction leading to compact column subtraction.



Partitioning to subtract with	Model using dienes and a place value mat.	Children to represent dienes using lines for 10's and dots or crosses for ones.	Expanded Column Subtraction leading to compact column method.
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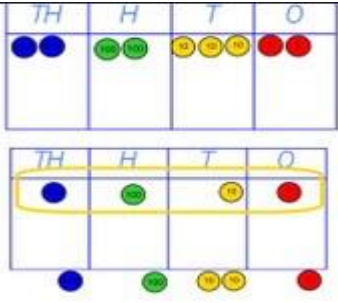
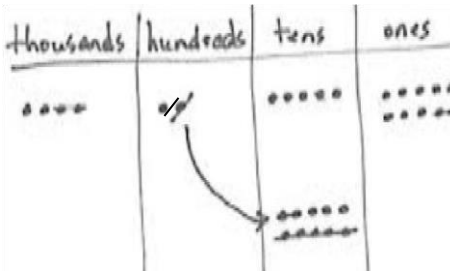
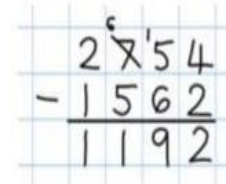
regrouping	<p>41 - 26 = 15</p>	<p>41 - 26 = 15</p>	$\begin{array}{r} 70 + 4 \\ - 20 + 7 \\ \hline \end{array}$ $\begin{array}{r} \overset{00}{70} + \overset{14}{4} \\ - 20 + 7 \\ \hline 40 + 7 \end{array}$ $\begin{array}{r} \overset{0}{7} \overset{14}{4} \\ - \overset{2}{2} \overset{7}{7} \\ \hline \overset{0}{4} \overset{7}{7} \end{array}$
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Year 3

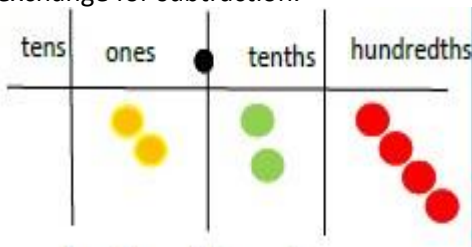
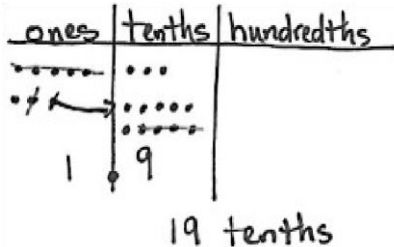
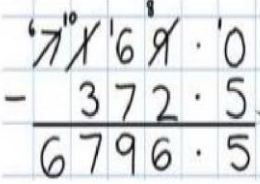
Strategies	Concrete	Pictorial	Abstract
Subtract numbers with up to three digits, using formal written methods of columnar addition.	<p>Use of place value counters.</p> <p>234 - 88 = 146</p>	<p>Represent the place value counters pictorially; remembering to show what has been exchanged.</p> <p>234 - 88 = 146</p>	<p>Expanded Column Subtraction leading to compact column method.</p> $\begin{array}{r} 700 + 40 + 1 \\ - 300 + 60 + 7 \\ \hline \end{array}$ $\begin{array}{r} \overset{000}{700} + \overset{130}{40} + \overset{11}{1} \\ - 300 + 60 + 7 \\ \hline 300 + 70 + 4 \end{array}$ $\begin{array}{r} \overset{0}{3} \overset{13}{7} \overset{11}{4} \\ - \overset{3}{3} \overset{6}{6} \overset{7}{7} \\ \hline \overset{0}{3} \overset{7}{7} \overset{4}{4} \end{array}$

Year 4

Strategies	Concrete	Pictorial	Abstract
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<p>Subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition).</p>	<p>Children continue to use place value</p>  <p>counters to subtract.</p> $2232 - 1121 = 1111$	<p>Draw representations using place value diagram.</p> 	<p>Compact column method to subtract.</p> 
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Year 5

Strategie s	Concrete	Pictorial	Abstract
<p>Subtract numbers with more than 4 digits.</p> <p>Subtract decimals with decimal places.</p>	<p>Introduce decimal place value counters and model exchange for subtraction.</p> 	<p>Draw representations using PV grid.</p>  <p>83 tenths - 64 tenths = 19 tenths</p> $\begin{array}{r} 83 \text{ tenths} - 64 \text{ tenths} \\ 8.3 - 6.4 \\ \underline{1.9} \end{array}$	<p>Children will subtract with decimal values, including mixtures of integers and decimals and aligning the decimal point. A zero will be used in any empty decimal place to aid understanding of what to subtract.</p> 

Year 6

Strategie s	Concrete	Pictorial	Abstract
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Subtract
more
complex
numbers
and decimal
values.

As year 5 using more complex numbers.

Multiplication

EYFS

Understanding of multiplication begins with practical work as children begin to work with groups of equal amounts. They learn to count in 2s, 5s and 10s, and this is then linked to practical problems, for example, counting pairs of children in the line, repeated printing of numicon shapes etc. Children begin to double numbers, initially with objects and later through addition.

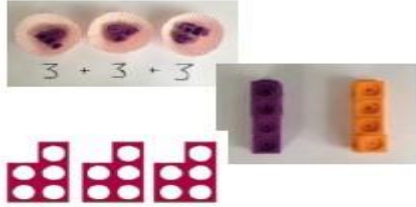
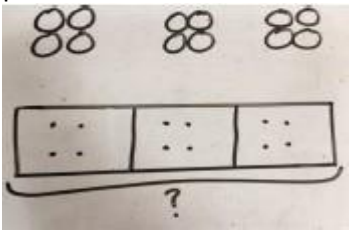
Children will begin to count objects in repeated sets:



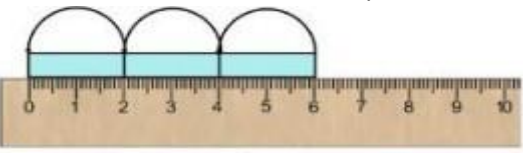
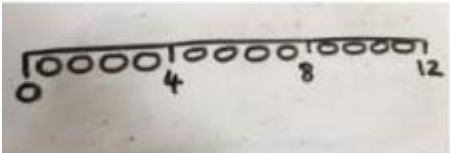
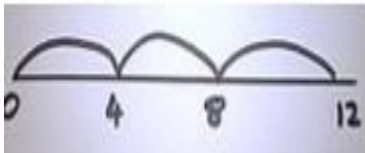

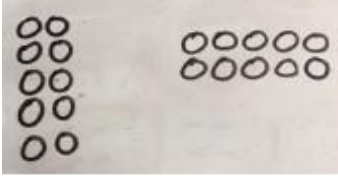
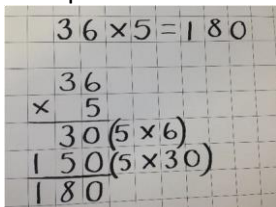
Using numicon to begin counting in repeated sets:



Year 1

Strategies	Concrete	Pictorial	Abstract
Repeated grouping/ repeated addition	Use manipulatives to make equal groups and  count the total.	Children to represent the practical resources in a picture. 	$3 \times 4 = 12$ $4 + 4 + 4 = 12$

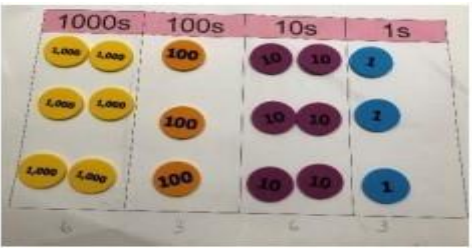
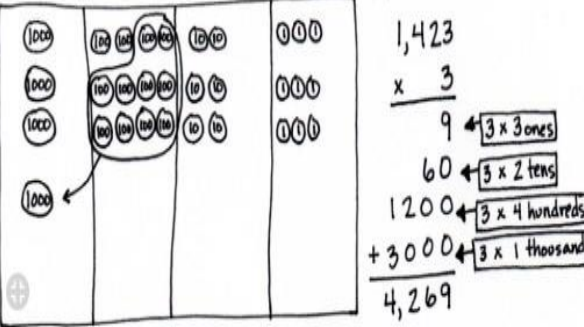
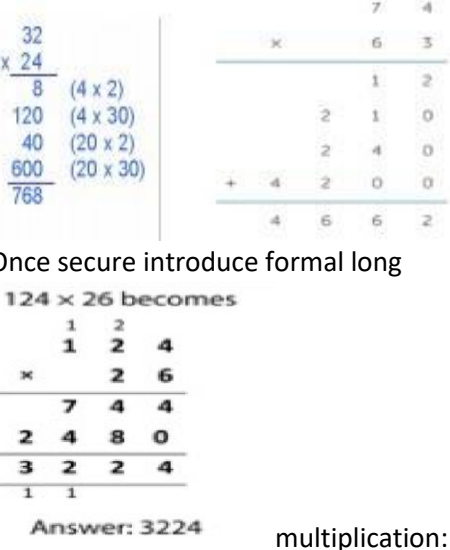
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Year 2			
Strategie s	Concrete	Pictorial	Abstract
Number line to show repeated addition.	Use Cuisenaire rods to show repeated groups above a ruler.  3 groups of 2	Represent this pictorially alongside a number line. 	Abstract number line showing three jumps of four.  $3 \times 4 = 12$
Showing multiplication as an array. Exploring its commutative properties.	Objects, cubes, counters and cuisenaire rods can be used to make arrays.  $5 \times 6 = 6 \times 5$	 Children to represent the arrays pictorially. $5 \times 2 = 2 \times 5$	Children to be able to use arrays to write a range of calculations e.g. $10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$
Year 3			
Strategie s	Concrete	Pictorial	Abstract
Multiplying two digit numbers by one digit.	Use dienes or place value counters to show how we are finding groups of a number. Then start by counting the ones, then tens. Then progress to making exchanges.	Represent the dienes or place value counters pictorially; remembering to show what has been exchanged.	Expanded short multiplication leading to short multiplication. 

	<p>6 x 23</p>		
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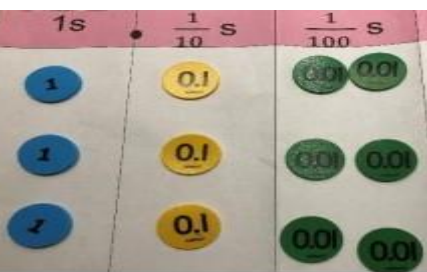
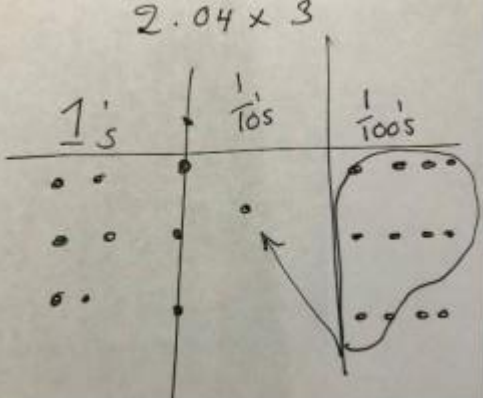
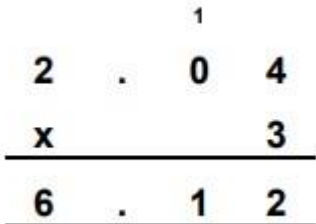
Year 4			
Strategies	Concrete	Pictorial	Abstract
<p>Multiplying two and three digit numbers by one digit.</p>	<p>Use dienes or place value counters to show how we are finding groups of a number. Then start by counting the ones, then tens and hundreds. Then progress to making exchanges.</p>	<p>Represent the dienes or place value counters pictorially; remembering to show what has been exchanged.</p>	<p>Expanded Short Multiplication leading to Short Multiplication:</p>

Year 5			
Strategies	Concrete	Pictorial	Abstract
<p>Multiply numbers up to 4</p>	<p>Pupils to use place value counters to represent 4 digit x 1 digit calculations.</p>	<p>Pupils to represent place value counters pictorially.</p>	<p>Consolidate short multiplication before moving on to expanded long multiplication:</p>

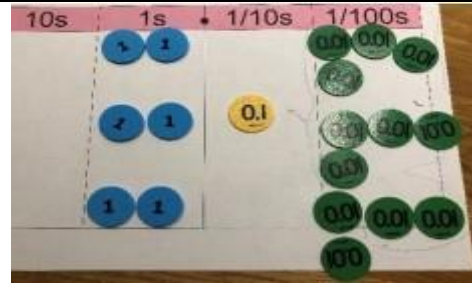
<p>digits by a one or two digit number.</p>	<p>$2,121 \times 3 =$</p> 		 <p>Once secure introduce formal long</p> <p>124 x 26 becomes</p> <p>Answer: 3224</p> <p>multiplication:</p>
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Year 6

Strategies	Concrete	Pictorial	Abstract
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<p>Multiply numbers up to 4 digits by a one or two digit number. (See Year 5)</p> <p>Multiplying decimals up to 2 places by a whole</p>	<p>Use place value counters to multiply decimals.</p>  <p>No exchange: 1.12×3</p> <p>With exchange: 2.04×3</p>	<p>Use pictorial representations.</p> 	<p>Formal method for multiplication.</p> 
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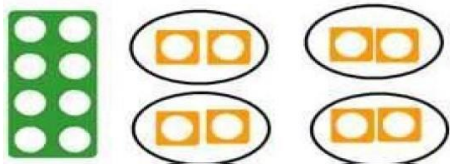
number.



Division

EYFS

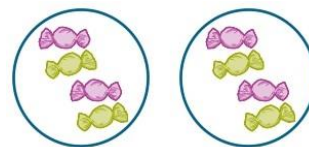
Children will engage in a wide variety of songs and rhymes, games and activities. In practical activities and through discussion they will begin to solve problems involving halving and sharing. The children will understand sharing as giving everyone the same amount. Make use of everyday situations e.g. sharing fruit out at snack time, in the role play area, sharing out objects in a practical way.



Share the apples between two people.



'Half the apples for me, half the apples for you.'



Year 1

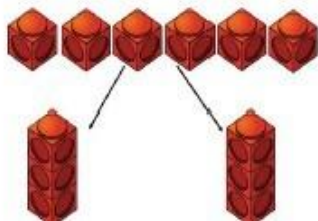
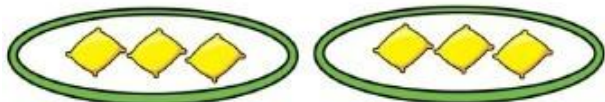
Strategies

Concrete

Pictorial

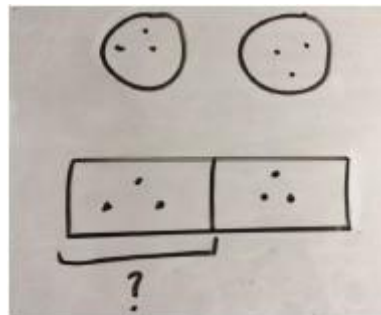
Abstract

Division as sharing.




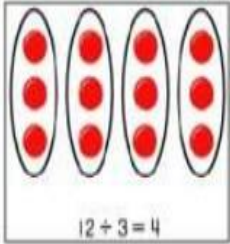
Sharing using a range of resources.

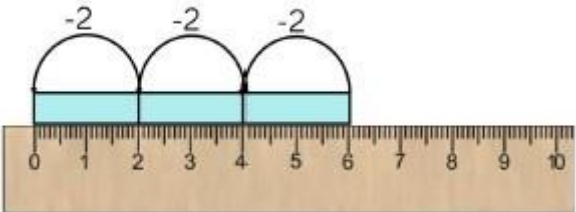
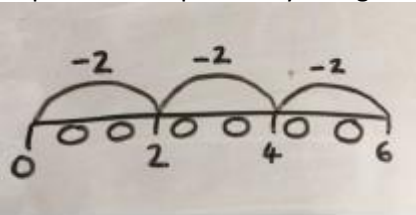
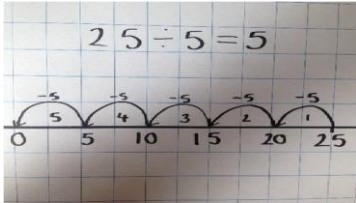
Represent the sharing pictorially.



$6 \div 2 = 3$

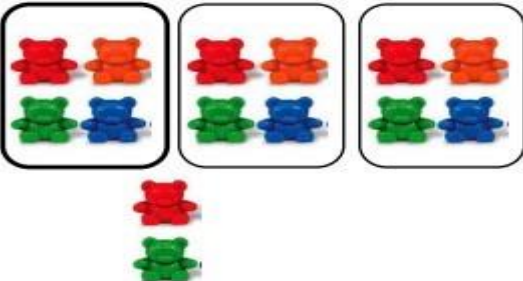

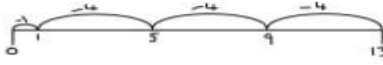
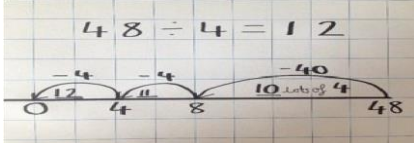
Year 2

Strategies	Concrete	Pictorial	Abstract
Showing division as an array. (division as grouping)	Numicon should be used to show how many groups of a number are in another number.  e.g. $6 \div 2 = 3$ "How many groups of 2 are there in 6?"	Children to represent the arrays pictorially.  $12 \div 3 = 4$	Children to be able to use arrays to write a range of calculations e.g. $12 \div 3 = 4$ $12 \div 4 = 3$ $4 \times 3 = 12$ $3 \times 4 = 12$
		This represents $12 \div 3$, posed as how many groups of 3 are in 12. Pupils should also know that the same array can represent $12 \div 4$ if grouped horizontally.	

Number line to show repeated subtraction. (division as grouping)	Use Cuisenaire rods to show repeated subtraction. $6 - 2$  $3 \text{ groups of } 2$ groups above a ruler.	Represent this pictorially alongside a number line. 	Abstract number line showing repeated groups subtracted.  $25 \div 5 = 5$
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Year 3

Strategies	Concrete	Pictorial	Abstract
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Dividing two digit numbers by one digit including with remainders.	Divide objects between groups and see how much is left over. 	Children to carry out grouping pictorially using dots or crosses. 	Abstract number line showing repeated groups subtracted and remainder using times tables facts to help them.  <p>Encourage children to take larger chunks using 'friendly' multiples of 2, 5 and 10.</p> 
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Year 4			
Strategies	Concrete	Pictorial	Abstract
Dividing two and three digit numbers using short division, including	Short division using place value counters to group. $615 \div 5 =$	Represent the dienes or place value counters pictorially; remembering to show what has been exchanged.	Children to represent the calculation using the short division scaffold:

remainder s.			$5 \overline{) 615}$
	<ol style="list-style-type: none"> 1. Make 615 with place value counters 2. How many groups of 5 hundreds can you make with 6 hundred counters? 3. Exchange 1 hundred for 10 tens. 4. How many groups of 5 tens can you make with 11 ten counters? 5. Exchange 1 ten for 10 ones. 6. How many groups of 5 ones can you make with 15 ones? 		$6 \overline{) 284} \begin{array}{l} 47 \\ r2 \end{array}$

Year 5

Strategies	Concrete	Pictorial	Abstract
Divide four digit numbers by one digit including remainders.	Continue to use place value counters as in year 4.	Continue to represent place value counters pictorially as in year 4.	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> $3192 \div 7 =$ $\begin{array}{r} 456 \\ 7 \overline{) 3192} \end{array}$ </div> <div style="text-align: center;"> $2940 \div 8 =$ $\begin{array}{r} 367r4 \\ 8 \overline{) 2940} \end{array}$ </div> </div> <p>Answers could also be given as remainders, decimals or fractions e.g. $2940 \div 8 = 367 \text{ r}4 = 367 \frac{1}{2} = 367.5$</p>

Year 6

Strategies	Concrete	Pictorial	Abstract
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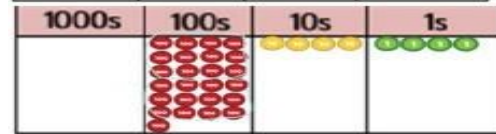
Divide four digit numbers by two digit numbers using long division.

Use Place value counters alongside the calculation.

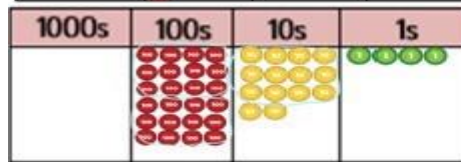
$$2544 \div 12$$



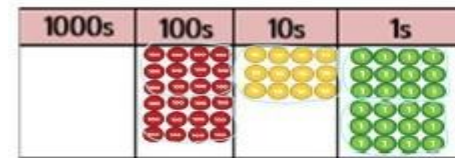
We can't group 2 thousands into groups of 12 so will exchange them.



We can group 24 hundreds into groups of 12 which leaves with 1 hundred.



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.



After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 1 \end{array}$$

$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$